Godel in the Mind

Introduction

Godel published his Incompleteness Theorem in 1931 [1]. The theorem implies that in any language defined solely by rules of syntax, we can always construct an undecidable proposition. An *undecidable proposition* is a statement akin to, "This statement is false." That is, it is a proposition whose truth or falsehood cannot be determined. Godel's theorem came as a blow to Logical Positivists. Prior to the publication of the theorem, Logical Positivists had claimed one could make a perfect model of the universe out of a combination of logic and a syntactically defined language. They took Godel's theorem to imply that their efforts were futile. They felt there could be no physical analog to undecidable propositions.

I claim we can readily conceive of physical events every bit as undecidable in our minds as Godel's propositions are undecidable in logical systems. That is to say, our contemplation of the universe can be frustrated by undecidable ideas in the same way mathematics is frustrated by undecidable propositions. Although undecidable propositions in mathematics may not be useful representations of undecidable ideas, the phenomenon of undecidability manifests itself similarly in both systems.

Some Potentially Undecidable Ideas

The first source of undecidable ideas that springs to my mind is to propose the existence of a time machine. H.G.Wells began writing stories about time machines at the end of the nineteenth century [2]. Recent papers in general relativity suggest ways in which time travel could be brought about [3]. Physicists have begun to wrestle with the paradoxes of time travel. Might we be able to go back in time and kill ourselves? Starting modestly, physicists studied a simple scenario in which a billiard ball flies through a time machine, travels back in time a few seconds, and collides with itself on the way into the machine [4]. Can the billiard ball knock itself away from the time machine entrance so as to prevent itself from passing through in the first place? They discovered that one cannot analyze the problem successfully by assuming the billiard ball goes through the hole irrespective of what it does afterwards. After thorough calculations, they found that it was physically impossible for the billiard ball ever to follow a trajectory through the time machine that would knock itself away from the entrance. Thus the billiard ball paradox appears to have been resolved.

Although time machines at first look promising as a source of undecidable ideas, physicists believe all such paradoxes can be resolved, even to the point of explaining how people cannot go back in time and kill themselves. Therefore I shall not use time-travel as a source of undecidable ideas. A truly undecidable idea must be as inevitably undecidable as is the proposition, "This statement is false." That is to say, undecidability must be one of its defining properties.

A Genuinely Undecidable Idea

Suppose there are a large number of stepping-stones in a lake, separated on average by a distance greater than I can cross with my most energetic leap. The distribution of the stones is not uniform, so almost all stones are closer to some stones than they are to others. Indeed, some of the stones are within leaping distance of other stones. I am standing upon one stone, and I would like to know which others are accessible to me by means of leaping from stone to stone, starting from the stone upon which I am standing. I consider each stone, hoping to decide whether or not it can be reached. I decide that some of the stones are not only mobile, but also telepathic. These stones know whether or not I have decided they are accessible, and are able to move on the basis of this knowledge. I entertain the possibility that one of these telepathic stones, which I shall refer to as the troublesome stone, has adopted a policy of moving away from its neighbors whenever I decide it is accessible, thus rendering itself inaccessible, and moving closer to its neighbors whenever I decide it is inaccessible, thus rendering itself accessible.

When I consider whether or not I can reach the troublesome stone, I find I cannot decide either way. Any time I decide such a stone can be reached, it is apparent to me that I am wrong, and whenever I decide the opposite, it is also apparent to me that I am wrong. The accessibility of the troublesome stone presents to the system of ideas and intuition operating in my mind the same dilemma of undecidability as one of Godel's undecidable propositions presents to a system of language and logic.

Godel pointed out that although a certain proposition may be undecidable in one logical system, its truth or falsehood within that system could readily be decided within another, or `meta-', system. The same may be said of the accessibility of the troublesome stone. It is undecidable in my mind, but in your mind, it is readily decidable. You know I can never reach a troublesome stone, for if I were to attempt to do so, it would move away so as to prevent me, and if I do not attempt to do so, I will not reach it anyway. Thus *you* can decide which stones are accessible to me, but *I* cannot.

Ways to deal with Undecidable Propositions and Ideas

Two solutions to the mental dilemma described above suggest themselves. The first is for me to deny the existence of the troublesome stone. The second is for me to assert that no one other than myself has absolute knowledge of my thoughts. Using the first solution, I allow for the existence of mobile, telepathic, stones, so long as they are not troublesome. Using the second solution, I simply deny the existence of anything that can read my mind. The same solutions may be applied to the mathematical dilemmas caused by undecidable propositions. In mathematics, the first solution amounts to a censure upon all undecidable propositions. The second solution amounts to a censure upon all propositions referring to truth (as it is defined within the system).

Both these solutions are inconvenient to the mathematician. A logical system is most useful if it is self-contained and unchanging. The first solution dictates that we eliminate undecidable propositions from our mathematical language as and when we find them, which means that our mathematical language will be evolving rather than unchanging. The second solution dictates that no mathematical proposition can refer to mathematical truth. This would require us to present mathematical proofs in another, meta-system, so our original system would not be self-contained.

Nevertheless, mathematicians must do something about undecidable propositions, and it appears they have adopted the former. Most mathematics text books do not mention undecidable statements, even though they make liberal use of proof by contradiction, a method of inference that is invalidated by the existence of undecidable propositions.

Conclusion

Just as undecidable propositions can be constructed in mathematical languages, undecidable ideas can be composed in our minds. Both are equally undecidable, and equally frustrating. The common-sense solution to the frustration caused by undecidable ideas is simply to ignore them. The same solution appears to have been adopted by mathematicians in response to Godel's formally undecidable propositions. Although we recognize that undecidable propositions are permitted by the syntax of our mathematical languages, in practical mathematicians ignore them. Effectively, such propositions are scratched from the list of legitimate propositions the moment they are discovered. Unsatisfactory though this solution may be to some, it has none the less been adopted, and appears to work well enough.

[1] Kurt Godel, On Formally Undecidable Propositions of Principia Mathematical and Related Systems I, reprinted in Godel's Theorem in Focus, S. G. Shanker, Routledge, 1991.
[2] H. G. Wells, The Time Machine.
[3] Astronomy, May 1995
[4] http://www.time-travel.com/whtitrip.htm